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EFFICIENT ALLOCATION OF RESOURCES

Tjalling C. Koopmans

P-116

7 December 1949

The RAND Corporation
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List of Changes in

Efficient Allocation of Resources

By Tjalling C. Koopmans

Page 1, title: Change footnote reference from * to 1.

after author's name: Change footnote reference from 1 to 2.

above line 1 of text: Insert section heading: INTRODUCTION

line 4: For "can they be separated" read "let us first ask ourselves whether they can be separated."

line 11: Insert "However," before "such".

line 13: For "production problems as" read "the potentialities of efficient production in the same way".

line 14: Insert "of production" after "factors".

line 18: Insert "indeed" before "possible".

line 19: For "In" read "With this understanding, therefore, we can in".

footnote *: Change * to "1". Insert "To be reprinted as Cowles Commission Paper, No. 52."

footnote 1: Change 1 to 2. Delete "several colleagues at the Cowles Commission, including". Insert "G. B. Dantzig, N. Georgescu-Roegen" after "Christ". For "Paper" read "An earlier version of this paper was". Include footnote in footnote 1.

page 2, line 2: Insert footnote reference 3 after "meetings".

line 4: Insert footnote reference 4 after "State".

line 6: For "operation" read "operations". Insert "economic theorist's model of" before competitive".

line 7: Delete "of economic theory".

line 9: Change footnote references 3, 4 to 5, 6.

line 12: Change footnote references 5-7, to 7-9.

line 16: For "It is sufficient" read "Efficiency can also be achieved".

page 2, Insert footnote 3 as follows: The meetings of the American Economic Association mentioned in footnote 1 were organized around the theme of "A Stocktaking of American Capitalism."

Renumber footnotes 2-7 to read 4-9.

page 3, line 11: Insert after "objective": "To maximize the military effectiveness or security obtained from given resources withdrawn from other uses, or, equivalently, to attain a required level of security at minimum cost in terms of resources withdrawn."

line 12: Change footnote reference from 8 to 10.

line 19: For "conducted too much in broad institutional terms" read "concerned too much with absolute institutional categories".

footnote: Change number from 8 to 10. Insert at end of footnote: "These articles have been reprinted, with some modifications, in Activity Analysis of Production and Allocation, Cowles Commission Monograph 13, Tjalling C. Koopmans, ed., New York; John Wiley & Sons, 1951."

page 4. Insert above line one the section heading: A MODEL OF PRODUCTION.

line 7: Change footnote 9 to 11.

line 10: For "Much substitution in production arises" read "Actually, much substitution in production, between alternative factors as well as between alternative products or product-factor combinations, arises".

line 11: Insert "discrete" before "processes".

line 12: Insert "continuous" before "variation"; for "in" read "within".

line 17: Change footnote 10 to 12.

footnotes: Change 9 and 10 to 11 and 12.

footnote 12, line 3: for "both to be included in a forthcoming monograph of the Cowles Commission" read "in Activity Analysis of Production and Allocation, op cit."; Insert at end of footnote: "Reprinted in Supplement to Econometrica, Vol. 17, July, 1949."

page 5, line 1: For "positive" read "nonnegative".

line 4: Insert after "outputs.": "All commodity flows and activity levels are constant through time."

line 13: For "are" read "may be".

line 17: Insert footnote reference 13 after "proportions".

Insert footnote 13 as follows: "For a given ratio of labor to land inputs (not corresponding to one of the points selected) the inputs of land and labor required to reach the isoquant level of output are slightly less in the continuous model than in the discrete model because in the continuous model labor can be applied uniformly in that ratio to all land involved, whereas in the (artificially restricted) discrete model the labor is applied in different ratios to two sectors of the land. However, the difference in inputs between the two models can be made arbitrarily small by selecting a sufficient number of basic points in the discrete model."

page 6, line 1: For "accomodates" read "can accommodate".

line 4: Delete comma after "alternatives" and insert "such".

line 5: Insert "various" before "goods".

line 10: Change footnote reference from 11 to 14.

line 15: For "feasible" read "possible."

line 17: For "accessible" read "attainable".

line 21: Change footnote reference from 12 to 15.

footnotes: renumber 11 and 12 to read 14 and 15.

footnote 15, line 3: For "and" read "while".

line 4: For comma before "zero" read "are"; For "input coefficients are" read "negative coefficients of the disposal activity are".

page 7, line 3: For "accessible" read "attainable".

line 4: For "accessible" read "attainable."

lines 6, 7: For "Instead of speaking of" read "As a synonym for"; For "we can also use the synonymous expression" read "we also use the expression".

page 7, line 10: Change footnote reference from 13 to 16; For "for" read "in".

line 12: For "does" read "will".

line 13: Change footnote reference from 14 to 17.

between lines 14 and 15: Insert section heading: EFFICIENCY PRICES.

lines 16, 17: Change footnote references from 14 and 15 to 18 and 19.

footnotes: Change footnote references from 13-15 to 16-18.

footnote 18, line 10: For "present analysis" read "present study".

page 8, line 2: For "in relation" read "relative".

line 4: Delete "entirely".

line 5: For "only elements underlying this price concept are" read "foundations on which this price concept is erected consist only of".

line 14: After "point" insert "in question".

line 18: Change (3) to read: "The prices of all final goods are positive."

between lines 18 and 19: Insert "(4) The prices of all primary goods are nonnegative."

line 19: For (4) read (5); For "or" read "of".

line 20: Change footnote 15 to 20.

line 21: For "on" read "as to".

footnotes: Change numbers from 16 and 17 to 19 and 20.

footnote 19: For "to be included in a forthcoming Cowles Commission Monograph" read "in Activity Analysis of Production and Allocation, op. cit."

page 9, line 1: For "accessible" read "attainable".

line 15: For "At those intersections" read "In an efficient point located on one of these intersections,".

lines 17, 18: For "In this case" read "In the special case of such an efficient point,".

page 10, line 5: For "against" read "for".

lines 12-15: Replace by "proposition). In that case, and only in that case, there is no way of using the new opportunity of exchange, whether in combination with variations in the levels of the other (productive) activities or not, so as to increase the net "output" of any final good without decreasing the net "output" of some other final good. Because of".

between lines 17, 18: Insert section heading: DECENTRALIZATION OF DECISIONS.

line 22: For "for attaining" read "that ensure".

line 24: For "nonnegative" read "positive".

page 11, line 1: For "is" read "may be".

line 4: For "No activity yielding" read "Any activity yielding".

line 5: For "engaged in" read contracted".

line 10: Change footnote reference from 18 to 21.

line 12: Insert period after "maintained" and delete rest of paragraph.

between lines 13 and 14: Insert following paragraph: "It should be emphasized that, if an inefficient state of resource allocation prevails initially, it is not claimed that adherence by all concerned to the rules stated would lead the mechanism to an efficient point, or even close to such a point, in a stated time interval. To establish such a claim would require a dynamic analysis resting on a more precise dynamic specification of the rules in question (by how much to expand a profitable activity, etc.). It is claimed only that adherence to the rules will perpetuate an efficient state once it has somehow come about. Thus, the idea of variation in the steering prices or in the technology to obtain other efficient points, must also be understood in the sense of comparative statics, not as a change through time." ¶ The rules that have been stated"

line 14: Delete "These rules" and run what follows into the above insert. For "stated" read "formulated". For "ange" read "Lange".

line 16: For "thus combine in one decision unit several" read "who thus represent in one decision unit a combination of several".

line 19: For "a composition" read "the composition".

line 25: Insert "process" before "managers".

line 26: Insert "process" before manager".

page 12, line 4: Change footnote reference from 19 to 22.

footnote: Change number of footnote from 19 to 22.

page 13, line 5: Insert after "vities.": "Physical depreciation of capital is taken into account, of course, as an input flow."

line 9: For "previously" read "usually".

line 10: Insert "both meaningful and" after "supposedly". For "knowledge is assumed" read "assumptions are made".

line 14: Insert "input-output" before "coefficients".

At end of article insert: Cowles Commission for Research in Economics

EFFICIENT ALLOCATION OF RESOURCES

by

Tjalling C. Koopmans*

The problems of welfare economics can be broadly arranged in two groups: the first group is concerned with most efficient allocation of resources in production, the second with most desirable distribution of commodities or of income. While these two problem areas can be distinguished, can they be separated? Do they not hang together particularly through the effect of the reward for human effort on the quantity and quality of that effort?

Any theory of optimal choice must specify what aspects of reality are accepted as given facts, what aspects are regarded as subject to choice, and what is the objective guiding such choice. The famous socialist maxim: "from each according to his ability, to each according to his needs," seems to assume that human effort is available in given quantity once needs are met. Such an assumption is not necessary to justify the separate study of allocative efficiency in production. We can study production problems as we study a demand curve or any other behavior schedule: Assuming that certain amounts of labor, land services and other factors are available in given quantities, how can we characterize efficient modes of production, according to some given criterion of efficiency? Such an inquiry can be useful even though considerations outside the sphere of production may further restrict the quantitative factor combinations that are possible.

* I am indebted to several colleagues at the Cowles Commission, including C. Christ, C. Hildreth, J. Marschak, S. Reiter, for valuable comments and suggestions.

In the present paper we concentrate entirely on the problem of efficient allocation in production. Much of the literature touching on this topic is concerned with the evaluation of alternative institutional or administrative forms of organizing production. This evaluation forms part of the grand debate on the merits of private or corporate enterprise versus a centrally directed economy -- a debate touching upon the broad theme of the present meetings — and important insights about out topic can be gained from this debate. The famous article by Enrico Barone on "The Ministry of Production in the Collectivist State"[1] emphasized the idea that an economy under centralized direction, to be efficient, should in most of its operation satisfy the same formal conditions as are satisfied by the competitive society of economic theory. This idea has been substantially accepted by all participants in the ensuing debate. The controversy was about methods of satisfying these conditions. Von Hayek[2] and Robbins[3] argued that it was impossible to impose these conditions by explicit calculation in one central office and by centralized administrative direction based on the results of such calculation. A number of writers including Dickinson[4], Lange[5], and Lerner[6], then took up the "Socialist" side of the debate, incorporating in their model of a socialist economy the relevant theoretical consecuences, but not the actual form, of competitive organization. These authors argued that, in

1. In "Collectivist Economic Planning," edited by F. von Hayek, London, 1935, see pp. 245-290.
2. F. von Hayek, "The Present State of the Debate," l.c. pp. 201-243, especially pp. 207-214.
3. L. C. Robbins, The Great Depression, London, 1934, see p. 151.
4. M. D. Dickinson, "The Economic basis of Socialism," Political Quarterly, Sept.-Dec., 1930.
5. O. Lange and F. Tailor, "On the Economic Theory of Socialism," Minneapolis, 1938.
6. A. P. Lerner, The Economics of Control, New York, 1946.

order to attain the objectives of a socialist economy centralized calculation is not necessary. It is sufficient if all managers of individual plants or industries respond to a price system applicable to the whole economy, in a manner prescribed by the following rules: the manager of any plant should produce any output or output combination at minimum cost, and the manager of any plant or industry should arrange for production at such a level as to equate price and marginal cost.

This all too brief survey shows that there was considerable adaptation of earlier notions of a socialist economy to the theorist's image of a competitive enterprise society, as the discussion went on. To remind us that the real world always offers a greater variety of problems than our attempts at theorizing have envisaged, a new contribution to the discussion has recently come from outside academic economics. M. K. Wood, a scientist and administrator, and G. B. Dantzig, a mathematician, both of the Department of the Air Force, were faced with the allocation problems of a widely ramified part of the military establishment. In this problem, the existing administrative structure provides no alternative to central direction of a complicated effort involving a large number of goods and services, all to be geared to one general objective. In this situation, as they report in two recent articles in Econometrica,[7] they revert to the method discarded by all participants

7. Marshall K. Wood and George B. Dantzig, "Programming of Interdependent Activities: I General Discussion," Econometrica, Vol. 17, July-October, 1949, pp. 193-199; and George B. Dantzig, "Programming of Interdependent Activities: II Mathematical Model," ibid., pp. 200-211.

in the debate who came after Barone: the actual collection of relevant technical information in one center and the calculation of an allocation program to serve as a basis of a large number of detailed directives. They see in the development of electronic computers a new possibility for this method, unforeseen in earlier phases of the discussion.

This interesting turn in the discussion shows, it seems to me, that the earlier discussions had been conducted too much in broad institutional terms encompassing the entire economy. Even in the capitalistic enterprise economy, there are many sectors where the guideposts of a competitive market are lacking, and explicit analysis of the allocation problem is needed. Another example may be added to that discussed by Wood and Dantzig. In determining the best pattern of routing of empty railroad cars, there are no market quotations placing differential prices on alternative geographic locations of cars. Present arrangements only permit this complicated problem to be handled by administrative direction.

In most of the present paper we shall therefore leave aside the question of the institutional arrangements under which allocative decisions are made. We wish to concentrate on the formal conditions for efficient use of resources, so as to leave the door open for later application within the plant, or to the individual firm, to public enterprises or adminstrative organs, to an industry, or to the economy as a whole. The main departure from previous analyses in welfare economics is in the adoption of a different model of production.[8] In

8. This model was first presented before the Madison meeting of the Econometric Society in August 1948, see "A Mathematical Model of Production," Econometrica, Vol. 17, No. 1, Jan. 1949, pp. 74-5. It is closely related to the model of von Neumann, see "A Model of General Economics Equilibrium," Review of Economic Studies, Vol. 13 and 14, 1945-47, pp.1-9, and to that of Dantzig, l. c.

this model, we shall not presuppose that marginal cost is necessarily known quantitatively to the manager of the individual production process, or even definable in terms of technological data available to him. Much substitution in production arises through shifts in the extent to which alternative processes are used, rather than through variation in factor combinations in the individual process. The purpose of the present model is to demonstrate that the possibility of such shifts is sufficient by itself to establish the concepts of marginal productivity and marginal cost where applicable, and a more general concept where these are not applicable. For simplicity, we consider a static model only.

We shall employ two basic concepts,* the commodity and the activity. Each commodity is assumed homogeneous and perfectly divisible. An activity, if carried out at a unit level, consists in the transformation of given quantities of some commodities into given quantities of other commodities, per unit of time. It is assumed that the level of any activity can be any positive multiple of the unit level, and that the commodity flows involved are the same multiple of those involved in the unit level of that activity. Negative flows represent inputs; positive flows, outputs.

It will be clear that this model rules out indivisibilities as well as increasing or decreasing returns to scale, and cannot be used in the analysis of any problems in which these phenomena are important elements. Apart from this specialization, the flexibility of the model described

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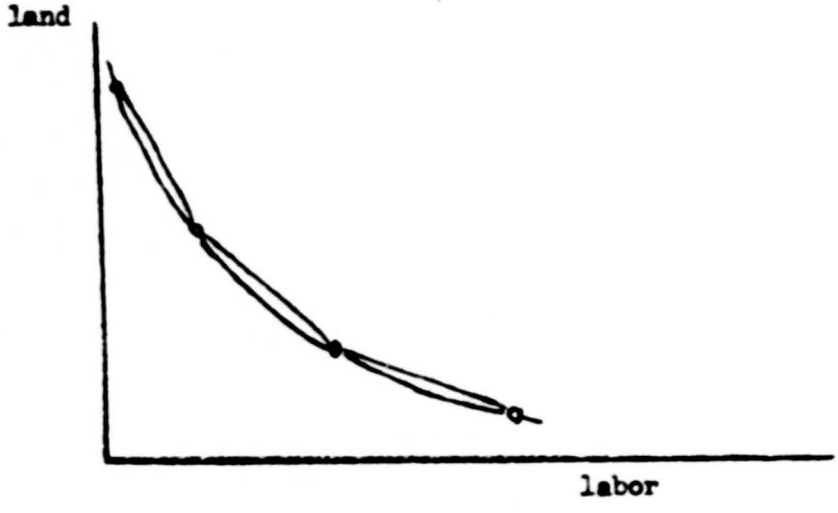
* For illustrative applications of these concepts, see Marshall K. Wood and George B. Dantzig, op. cit.; C. Hildreth and S. Reiter, "On the Choice of a Crop Rotation Plan," and T. C. Koopmans and S. Reiter, "A Model of Transportation," both to be included in a forthcoming monograph of the Cowles Commission; see also T. C. Koopmans, "Optimal utilization of the transportation system," Proceedings of the International Statistical Conference held in Washington, D. C., September 6-18, 1947, Vol. 5, 1949.

needs to be emphasized. Cases where continuous substitution within one productive process is possible can be approximated as closely as desired, at least for purposes of theory, by introducing a larger number of activities. If wheat production depends continuously on the quantities of land and labor, one may select a sufficient number of quantitative ratios in which land and labor are combined, and let each of these define a possible wheat-producing activity. With reference to a given isoquant curve in the land-labor-plane, this means the selection of a number of points on the curve (see diagram). Other points on the isoquant curve can then be



approximated by combining two adjacent selected activities in suitable proportions. A similar device can be applied if more than two factors are continuous substitutues. Thus, while the model accommodates cases of continuous substitution, it is especially designed to include those cases where substitution consists in relative quantitative shifts between discrete alternatives, as we see so often in industry.

We shall classify the goods and services into three categories, which may of course be defined differently in different applications of

the model. Primary goods are those which flow into production from nature, or from outside the sector of the economy considered, at a rate which by assumption cannot exceed given availability limitations. Final goods are those produced goods which are desired for purposes of consumption or delivery outside of the sector of the economy studied.[9] Intermediate goods are those produced goods which are not wanted in themselves, that is, for any purpose other than their use as inputs to further activities of which the ultimate purpose is the production of final goods.

To any set of nonnegative levels of the activities corresponds what we shall call a (technologically) feasible set of net output flows of all commodities. We shall call the set of activity levels, and also the corresponding set of net commodity flows, (economically) accessible if (a) the net flows of all final commodities are nonnegative (because we cannot draw directly on nature to fill a deficit), if (b) the net flows of all primary goods are nonpositive (because we have no use for the surplus) and stay within the availability limitations, and if (c) the net flows of all intermediate goods are zero,[10] (because both considerations adduced in parentheses above apply).

We now come to the concept of productive efficiency upon which all subsequent analysis builds forth. An accessible set of commodity

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9. To obtain a clear separation between primary and final goods, we shall regard direct consumption of goods available in nature, such as drinking water from a brook, as an activity converting the natural resource "water" (input coefficient -1) into the consumption good "drinking water" (output coefficient +1). Similarly, leisure is the output of a recreation activity of which the input is that part of labor which is not used as input in other activities.

10. It might be thought that waste products are intermediate goods with positive net output. However, if their disposal is costless, we can maintain the zero net output condition by introducing a disposal activity with input coefficient -1, and all other coefficients, zero. If disposal ties up resources, additional input coefficients are called for, and the zero net output requirement is essential.

flows, as well as any set of activity levels giving rise to it, is called efficient if there is notother feasible set of commodity flows in which all flows are at least as large as the corresponding flows in the original set, while at least one is actually larger. Instead of speaking of an efficient set of commodity flows, we can also use the synonymous expression: an efficient point in the commodity space. In general, the notion of the set of all efficient points corresponds to the notion of a general transformation function, discussed by Lange[11] and others. However, for certain models, depending on the number of activities and the values of their technological coefficients, the efficient point set will not possess a sufficient number of dimensions to make the notion of a transformation function applicable.* The efficient point set therefore constitutes the more general concept of the two.

We shall now give a number of conclusions that can be derived by mathematical analysis** from the model that has been formulated, while referring to another publication[12] for proofs and more detailed explanations. The first conclusion is, that where the efficient point set

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11. Oscar Lange, "The Foundations of Welfare Economics," Econometrica, Vol. 10, July-October, 1942, pp. 215-228.

* This may come about because the number of activities is too small in relation to the number of final goods, or because a small number of activities jointly hold a position of technical superiority over all other activities for all conceivable compositions of demand.

** The type of mathematical analysis involved is rather different from that found in most mathematical discussions of production theory, largely because of the role played by linear inequalities such as arise from the nonnegative character of activity levels and from the availability limits on primary goods. The theory of convex sets, particularly convex polyhedral cones, is drawn upon. There are many points of contact with a model constructed by von Neumann, see "A Model of General Economic Equilibrium," Review of Economic Studies, Vol. XIII (1) 1945-46. An important difference is that in von Neumann's model the efficiency of allocation comes out at the end as a by-product of an analysis concerned mainly with an existence theorem, while in the present analysis efficiency of allocation is made the central theme of analysis.

is indeed representable by a general transformation function, there are constant or decreasing returns in the output of any one final good, in relation to increases in the input limit (availability limit) on any one primary factor.

Further propositions introduce a price concept which is entirely independent of the notion of a market. The only elements underlying this price concept are the technological data (input-output coefficients of all activities) and the requirement of efficiency.

The first price proposition says that for each efficient set of commodity flows there exists an associated set of prices for all commodities, with the properties listed below. To formulate these properties we define the concept of the profitability of an activity as the aggregate value, at the prices in question, of the outputs associated with the unit level of that activity, minus the aggregate value of the corresponding inputs. The properties of the set of prices, associated with the efficient point, are the following:

(1) No activity has a positive profitability.

(2) Any activity carried out at a positive level to attain the efficient set of commodity flows has a zero profitability.

(3) The prices on all final and primary goods are nonnegative.

(4) The prices on all primary goods whose net input does not reach the availability limit are zero.*

It will be noted that no statement is made on the signs of the prices of intermediate goods. Indeed, negative prices will arise for waste

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12. T. C. Koopmans, "Analysis of Production as an Efficient Combination of Activities," to be included in a forthcoming Cowles Commission Monograph.

* These goods are therefore properly called free goods with reference to the efficient point in question.

products, the disposal of which uses up positively priced goods.

The second price proposition states that the converse is also true. An accessible point with which a set of prices with the listed properties can be associated is an efficient point.

It should be mentioned that the set of prices associated with a given efficient set of commodity flows is not necessarily unique. In the case where a general transformation function exists, the set of prices will be unique almost everywhere on the hypersurface represented by that function. If the set of prices is unique, ratios of the prices of final and/or primary goods can be interpreted as marginal rates of substitution between goods. The substitution in question arises from such variations in the activity levels as to leave all commodity flows constant except those of the two goods between which substitution is considered - while maintaining efficiency in the activity combination before and after variation. These rates of substitution are applicable to finite (as distinct from infinitesimal) variations in the commodity flows involved, but often only with finite limits of variation. This comes about because the hypersurface representing the transformation function consists of sections of hyperplanes joined at their intersections. At those intersections marginal rates of substitution are generally different for increases and for decreases in the net output of any one of the two commodities in question. In this case more than one set of associated prices (to be precise: an infinity of set of prices) satisfies the requirements stated.

It may be restated that the price concept established does not in any way presuppose the existence of a market or of exchanges of commodities between different owners. The price concept is found to be a mathematical

consequence of an efficient choice of activity levels. In the important case in which the activities engaged in are sufficient in number and variety to lead to a unique solution of the condition (2) that their profitabilities be zero, the prices have already been interpreted as technological rates of substitution under efficient allocation. An additional interpretation, not thus limited in its applicability, is derived from the following third price proposition.

Let us consider a given efficient set of commodity flows, and let us add to the technology a number of exchange activities, defined by means of an arbitrarily selected set of prices, as follows: We shall imagine that, through contacts with another economy, any commodity can be exchanged against any other commodity at a price ratio computed from the selected set of prices. Then our proposition specifies the conditions under which, after the opportunity to engage in these exchange activities has been provided, the original set of commodity flows will still be efficient. This original set of flows remains efficient if and only if the prices selected to define the exchange activities are at the same time a set of prices associated with the original set of flows, (in the sense of the first price proposition). In that case, and only in that case, there is no way to increase the net "output" of any final good, without decrease in the net "output" of some other final good, by using the new possibility of exchange, possibly in combination with variations in the levels of the other (productive) activities. Because of this proposition a set of prices associated with an efficient point can also be called a corresponding set of efficiency prices.

With the help of these formal propositions, we shall now consider, although still in a rather abstract fashion, some institutional

arrangements under which efficient allocation may be attained. The propositions stated, and in particular the profitability properties (1) and (2) of the efficiency prices, strongly suggest mechanisms for attaining efficiency by decentralization of decisions concerning activity levels, mechanisms which are similar to those referred to above. Let us assume, in the first allocation model, that a set of nonnegative prices on final commodities is prescribed, either administratively by some central authority, or as market prices reflecting a balance of preferences of consumers weighted by the income distribution. These may be called steering prices in that through their variation it is possible to steer the allocation of resources to the production of alternative efficient sets of commodity flows. Let the level of each activity and the prices of primary and intermediate goods be determined by a bidding process governed by the following rules: No activity yielding a negative profit is to be engaged in. Any activity yielding a zero profit is to be maintained at a constant level. Any activity yielding a positive profit is to be expanded, if necessary by bidding up prices of its input commodities. Behavior according to these rules could either be induced by administrative authority binding the action of managers of individual activities, or it could result from a competitive market structure where each activity is engaged in by many independent entrepreneurs.* The second price proposition now implies that efficiency, once attained in such an administrative or market structure, is maintained (provided there is no change in technology or in the steering prices).

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* We are, of course, concerned with the logical content of the rules only, not with the question whether and how compliance by managers or entrepreneurs can be secured.

These rules are not yet equivalent to those stated by Lange, Lerner and others. The latter rules apply to plant managers who control more than one activity in our sense of the term, and thus combine in one decision unit several, possibly many, of our fictitious activity managers. However, the Lange-Lerner model is closely approached in the following second allocation model: Define as a process a set of activities controlled by one manager. Define as a composition of a process the set of activity levels selected by its manager. Now, the attainment of efficiency in the economy as a whole has as prerequisites (A) the attainment of efficiency by each process manager within the set of activities controlled by him, and in particular, (B) the selection of such an efficient activity composition by each process manager that an associated set of efficiency prices exists which is the same for all managers. Efficiency for the economy as a whole, once attained, will be maintained if each manager behaves according to the following rules: Choose only from those sets of activity levels that correspond to an efficient point within your process. If for all such points the profit on the entire process is zero, discontinue all activity. If you are in a point of nonnegative profit on the process, attempt to raise* your profit-at-the-given-prices by varying the composition of the process. If you are in a point of zero profit, and there is no increase in profit possible by variation of activity levels, continue all activities at the same level. If your attempt to raise profit-at-given-prices leads to a rise in prices of certain input commodities, determine your further action in the light of the new price situation.

The reader will have realized that behavior according to these rules

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* If the profitability is initially positive, this can always be done by proportional increases in the levels of all activities in the process.

presupposes a knowledge, on the part of each process manager, of the efficient point set that can be constructed on the basis of those activities involved in the process that he controls. Also, behavior according to these rules is equivalent, in its outcome, to profit maximization under perfect competition.

Comparing the results of the above analysis with the earlier discussions referred to above, it appears that in one sense we have made more limited assumptions. We have ruled out both indivisibilities and increasing or decreasing returns to scale. As a result of that limitation we have obtained a stronger proposition. The rules on the allocation mechanism stated have been found to be not only necessary, but also sufficient for an efficient use of resources.

On the other hand, because of our simple assumptions we have not yet reached the problems arising from discrepancies between average and marginal cost, problems which have received considerable emphasis in earlier discussions.